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Project: Power Bill: Residential Rates

Company: Alabama Power

Objectives:

1. Calculate the summer power bill of the residents of the State of Alabama within each range of specific power usage manually using Arithmetic method.
2. Write a piece-wise function of the residential rates.
3. Recalculate the **same** power bill of the residents of the State of Alabama within each range of specific power usage algebraically using the Piece-wise Function method.

Information:

Billing Months: June - September	
Base Charge: \$14.50	
Charge Rate (¢/kWh)	**Power Usage** (kWh)
12.233	up to 1000
12.4859	over 1000

Numbers to be Tested: 0 kWh, 500 kWh, 1000 kWh, 1500 kWh

Arithmetic Method:

For the Arithmetic Method section, we'll be calculating the cost of the power bill by taking charge rates from the table in the Information section and multiplying them by the amount of power usage indicated by the values in the Numbers to be Tested section. To make the calculations easier, charge rates from the table expressed in ¢/kWh will be converted to \$/kWh by multiplying the ¢/kWh value by $\frac{\$1}{100\text{ ¢}}$. The following two calculations show this conversion, and the remaining calculations in this section are the power bill calculations.

$$\begin{aligned}
\frac{12.233\text{ ¢}}{\text{kWh}} \cdot \frac{\$1}{100\text{ ¢}} &= \frac{12.233\text{ ¢} \cdot \$1}{100\text{ ¢} \cdot \text{kWh}} \\
&= \frac{12.233 \cdot \$1}{100 \cdot \text{kWh}} \\
&= \frac{0.12233 \cdot \$1}{\text{kWh}} \\
&= \$0.12233/\text{kWh}
\end{aligned}$$

$$
\begin{aligned}
\frac{12.4859\ \text{¢}}{\text{kWh}} \cdot \frac{\$1}{100\ \text{¢}} &= \frac{12.4859\ \text{¢} \cdot \$1}{100\ \text{¢} \cdot \text{kWh}} \\
&= \frac{12.4859 \cdot \$1}{100 \cdot \text{kWh}} \\
&= \frac{0.124859 \cdot \$1}{\text{kWh}} \\
&= \$0.124859/\text{kWh}
\end{aligned}
$$

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0 kWh
Base Charge = \$14.50
Charge Rate = \$0.12233/kWh
Actual Power Usage = 0 kWh

Cost = Base Charge + Charge Rate x Actual Power Usage

$$
\begin{aligned}
\text{Cost} &= \$14.50 + \$0.12233/\text{kWh} \cdot 0\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233}{\text{kWh}} \cdot 0\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233 \cdot 0\ \text{kWh}}{\text{kWh}} \\
&= \$14.50 + \$0.12233 \cdot 0 \\
&= \$14.50 + \$0.00 \\
&= \$14.50
\end{aligned}
$$

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500 kWh
Base Charge = \$14.50
Charge Rate = \$0.12233/kWh
Actual Power Usage = 500 kWh

Cost = Base Charge + Charge Rate x Actual Power Usage

$$
\begin{aligned}
\text{Cost} &= \$14.50 + \$0.12233/\text{kWh} \cdot 500\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233}{\text{kWh}} \cdot 500\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233 \cdot 500\ \text{kWh}}{\text{kWh}} \\
&= \$14.50 + \$0.12233 \cdot 500 \\
&= \$14.50 + \$61.165 \\
&= \$75.665 \\
&\approx \$75.67
\end{aligned}
$$

1000 kWh
Base Charge = \$14.50
Charge Rate = \$0.12233/kWh
Actual Power Usage = 1000 kWh

Cost = Base Charge + Charge Rate x Actual Power Usage

$$\begin{aligned}
\text{Cost} &= \$14.50 + \$0.12233/\text{kWh} \cdot 1000 \text{ kWh} \\
&= \$14.50 + \frac{\$0.12233}{\text{kWh}} \cdot 1000 \text{ kWh} \\
&= \$14.50 + \frac{\$0.12233 \cdot 1000 \text{ kWh}}{\text{kWh}} \\
&= \$14.50 + \$0.12233 \cdot 1000 \\
&= \$14.50 + \$122.33 \\
&= \$136.83
\end{aligned}$$

For the previous three calculations, the actual power usage has been less than or equal to 1000 kWh. This means the same rate (12.233 ¢/kWh) has been used. For the following calculation, a power usage over 1000 kWh, it will be necessary to calculate the cost up to 1000 kWh as in previous calculations. However, it will then be necessary to use the second rate (12.4859 ¢/kWh) for all power usage over 1000 kWh.

Since the total usage is 1500 kWh and the rate changes at 1000 kWh, this will require subtracting 1000 kWh from the total 1500 kWh, leaving 500 kWh. The remaining 500 kWh is the power usage over 1000 kWh and will use the second, higher rate, and be added to the cost calculated for the usage up to 1000 kWh.

1500 kWh Base Charge = \$14.50
Charge Rate = \$0.124859/kWh
Actual Power Usage = 1500 kWh
Power Usage Up to 1000 kWh = 1000 kWh
Power Usage Over 1000 kWh = 1500 kWh - 1000 kWh = 500 kWh

$$\begin{aligned}
\text{Cost} = \text{Base Charge} \\
&+ \text{Charge Rate Up to 1000 kWh x Power Usage Up to 1000 kWh} \\
&+ \text{Charge Rate Over 1000 kWh x Power Usage Over 1000 kWh}
\end{aligned}$$

$$
\begin{aligned}
\text{Cost} &= \$14.50 + \$0.12233/\text{kWh} \cdot 1000 \text{ kWh} + \$0.124859/\text{kWh} \cdot 500 \text{ kWh} \\
&= \$14.50 + \frac{\$0.12233}{\text{kWh}} \cdot 1000 \text{ kWh} + \frac{\$0.124859}{\text{kWh}} \cdot 500 \text{ kWh} \\
&= \$14.50 + \frac{\$0.12233 \cdot 1000 \text{ kWh}}{\text{kWh}} + \frac{\$0.124859 \cdot 500 \text{ kWh}}{\text{kWh}} \\
&= \$14.50 + \$0.12233 \cdot 1000 + \$0.124859 \cdot 500 \\
&= \$14.50 + \$122.33 + \$184.7595 \\
&= \$136.83 + \$184.7595 \\
&= \$199.2595 \\
&\approx \$199.26
\end{aligned}
$$

Piece-wise Function:

In the Arithmetic Method section, we established how to manually calculate the cost for a monthly power bill for a customer of Alabama Power. Here, we will develop a more concise, algebraic function called a piece-wise function in order to calculate the same values. In order to develop the function, we will first define the variables and equations necessary to define the problem. Instead of performing the calculations manually at this point, however, we'll simplify the equations and combine them, as necessary, for concision. Lastly, we will express those simplified equations as a part of a single piece-wise function.

The total cost, C, for a monthly power bill for an Alabama Power residential customer is given by the equation

$$C = b + p,$$

where b is the base charge and p is the power usage charge. According to the table in the Information section, the base charge, b, is a constant \$14.50.

Again, from the table in the Information section, we know that the power usage charge is calculated using a given charge rate in cents per kilowatt hour (¢/kWh) for the actual number of kilowatt hours used (kWh). Given this relation, the power usage charge, p, can be calculated using the equation

$$p = r \cdot u,$$

where r is the given charge rate in cents per kilowatt hour and u is the total amount of power used in kilowatt hours.

The value of r, however, is determined by the amount of power used, so these equations are incomplete. Specifically, if the customer used up to 1000 kWh, then $r = 12.233$ ¢/kWh and therefore,

$$p_1 = r_1 \cdot u_1, \tag{1}$$

where p_1 is the power usage charge for the case where $u \leq 1000$, r_1 is the charge rate for the case where $u \leq 1000$ and u_1 is a $u \leq 1000$. For the case where $u > 1000$, $r = 12.4859$ ¢/kWh. This r, however, only applies to the usage over 1000 kWh. So, for a $u > 1000$ kWh, it's necessary to "calculate p twice," so to speak. In other words, the power usage charge must be calculated for the kilowatt hours used up to 1000 kWh, then again for the kilowatt hours over 1000 kWh, and the resulting two calculations must be summed.

The equation for the case where $u > 1000$ kWh can be expressed as

$$p_2 = r_2 \cdot u_2,$$

where r_2 is the charge rate for $u > 1000$ kWh, and u_2 is the total power usage less u_1. So, p becomes

$$p = p_1 + p_2.$$

Substituting p into the cost equation yields

$$C = b + (p_1 + p_2) = b + p_1 + p_2.$$

As in the Arithmetic Method section, we'll express the charge rates as dollars per kilowatt hour instead of cents per kilowatt hour to simplify the arithmetic for the remainder of this section. We're now ready to begin developing our piece-wise function.

The two cases that need to be accounted for are when $u \leq 1000$ and when $u > 1000$. Since the Alabama Power rate document doesn't mention negative kWh usage (from, for example, solar panels), we can further restrict the $u \leq 1000$ case to $0 \leq u \leq 1000$. So, substituting in our latest cost equation for both cases, the first instance of the cost function, $C(u)$, can be expressed as

$$C(u) = \begin{cases} b + p_1 + p_2; & 0 \leq u \leq 1000 \\ b + p_1 + p_2; & u > 1000 \end{cases}.$$

Since we know the charge rate for $0 \leq u \leq 1000$ kWh is \$0.12233/kWh and that u_1 will be u because $u \leq 1000$, we can update p_1 to be

$$p_1 = \$0.12233/\text{kWh} \cdot u.$$

We also know that p_2 will be 0 for the first case due to there being no power usage over 1000 kWh

$$\begin{aligned} p_2 &= r_2 \cdot u_2 \\ &= r_2 \cdot 0 \\ &= 0, \end{aligned}$$

so our cost function becomes

$$C(u) = \begin{cases} \$14.50 + \$0.12233/\text{kWh} \cdot u; & 0 \leq u \leq 1000 \\ b + (p_1 + p_2); & u > 1000 \end{cases}.$$

Since we know that $u_2 = u - 1000$ and the charge rate for the $u > 1000$ case is \$0.124859/kWh, p_2 after simplification becomes

$$\begin{aligned}
p_2 &= \$0.124859/\text{kWh} \cdot (u - 1000 \text{ kWh}) \\
&= \$0.124859/\text{kWh} \cdot u - \$0.124859/\text{kWh} \cdot 1000 \text{ kWh} \\
&= \$0.124859/\text{kWh} \cdot u - \frac{\$0.124859}{\text{kWh}} \cdot 1000 \text{ kWh} \\
&= \$0.124859/\text{kWh} \cdot u - \frac{\$0.124859 \cdot 1000 \text{ kWh}}{\text{kWh}} \\
&= \$0.124859/\text{kWh} \cdot u - \$0.124859 \cdot 1000 \\
&= \$0.124859/\text{kWh} \cdot u - \$124.859.
\end{aligned}$$

For the $u > 1000$ case, we know that u_1 will be 1000 kWh, since that is the max. for the $0 \leq u \leq 1000$ case. This makes p_1 for the $0 \leq u \leq 1000$ case

$$\begin{aligned}
p_1 &= \$0.12233/\text{kWh} \cdot 1000 \text{ kWh} \\
&= \frac{\$0.12233}{\text{kWh}} \cdot 1000 \text{ kWh} \\
&= \frac{\$0.12233 \cdot 1000 \text{ kWh}}{\text{kWh}} \\
&= \$0.12233 \cdot 1000 \\
&= \$122.33
\end{aligned}$$

With p_1 and p_2 known and the base charge of \$14.50, the expression for the $u > 1000$ case after simplification, becomes

$$\begin{aligned}
b + p_1 + p_2 &= \$14.50 + \$122.33 + \$0.124859/\text{kWh} \cdot u - \$124.859 \\
&= \$136.83 + \$0.124859/\text{kWh} \cdot u - \$124.859 \\
&= \$11.971 + \$0.124859/\text{kWh} \cdot u.
\end{aligned}$$

Finally, our fully developed piece-wise function becomes

$$C(u) = \begin{cases} \$14.50 + \$0.12233/\text{kWh} \cdot u; & 0 \leq u \leq 1000 \\ \$11.971 + \$0.124859/\text{kWh} \cdot u; & u > 1000 \end{cases}.$$

Piece-wise Function Method:

With our piece-wise function complete, we can proceed with testing it against the same values as were tested in the Arithmetic section. For each value to be tested, the piece of the function to use will depend on where the usage falls within the domain indicated by the function.

For example, if the value is 100 kWh, the first case ($\$14.50 + \$0.12233/\text{kWh} \cdot u$) will be used; for a use higher than 1000 kWh, say 1600 kWh, the second case ($\$11.971 + \$0.124859/\text{kWh} \cdot u$) will be used. All that needs to be done after picking the appropriate case is substituting the power usage value for u and evaluating the resulting function.

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0 kWh

Piece-wise Function Case $= \$14.50 + \$0.12233/\text{kWh} \cdot u$
$u = 0$ kWh

$$\begin{aligned}
C(0\ \text{kWh}) &= \$14.50 + \$0.12233/\text{kWh} \cdot 0\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233}{\text{kWh}} \cdot 0\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233 \cdot 0\ \text{kWh}}{\text{kWh}} \\
&= \$14.50 + \$0.12233 \cdot 0 \\
&= \$14.50 + \$0.00 \\
&= \$14.50
\end{aligned}$$

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500 kWh

Piece-wise Function Case $= \$14.50 + \$0.12233/\text{kWh} \cdot u$
$u = 500$ kWh

$$\begin{aligned}
C(500\ \text{kWh}) &= \$14.50 + \$0.12233/\text{kWh} \cdot 500\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233}{\text{kWh}} \cdot 500\ \text{kWh} \\
&= \$14.50 + \frac{\$0.12233 \cdot 500\ \text{kWh}}{\text{kWh}} \\
&= \$14.50 + \$0.12233 \cdot 500 \\
&= \$14.50 + \$61.165 \\
&= \$75.665 \\
&\approx \$75.67
\end{aligned}$$

1000 kWh

Piece-wise Function Case $= \$14.50 + \$0.12233/\text{kWh} \cdot u$

$u = 1000$ kWh

$$\begin{aligned}
C(1000\text{ kWh}) &= \$14.50 + \$0.12233/\text{kWh} \cdot 1000\text{ kWh} \\
&= \$14.50 + \frac{\$0.12233}{\text{kWh}} \cdot 1000\text{ kWh} \\
&= \$14.50 + \frac{\$0.12233 \cdot 1000\text{ kWh}}{\text{kWh}} \\
&= \$14.50 + \$0.12233 \cdot 1000 \\
&= \$14.50 + \$122.33 \\
&= \$136.83
\end{aligned}$$

1500 kWh

Piece-wise Function Case $= \$11.971 + \$0.124859/\text{kWh} \cdot u$

$u = 1500$ kWh

$$\begin{aligned}
C(1500\text{ kWh}) &= \$11.971 + \$0.124859/\text{kWh} \cdot 1500\text{ kWh} \\
&= \$11.971 + \frac{\$0.124859}{\text{kWh}} \cdot 1500\text{ kWh} \\
&= \$11.971 + \frac{\$0.124859 \cdot 1500\text{ kWh}}{\text{kWh}} \\
&= \$11.971 + \$0.124859 \cdot 1500 \\
&= \$11.971 + \$187.2885 \\
&= \$199.2595 \\
&\approx \$199.26
\end{aligned}$$

Citations (APA):

Alabama Power. (n.d.). Rate FD family dwelling- residential service. RATE FD - FD.pdf. Retrieved February 18, 2023, from `https://www.alabamapower.com/content/dam/alabama-power/pdfs-docs/Rates/FD.pdf`

Chukwuemeka, S. (n.d.). Piecewise functions. Piecewise Functions. Retrieved February 18, 2023, from `https://piecewise-functions.appspot.com/#studentProjectPowerBill`